Summary
Curved bridges have become a major component of highway systems in recent years. Elevated freeways and multi level interchange are very common in densely populated areas and could hardly be constructed without curved bridges. Usually, these bridges are of cellular cross section so that the high torsional moment due to curvature can be resisted economically.

To date, methods of analysis and design have been very approximate, but because of the large number of curved bridges being constructed everywhere, refined methods of analysis are desirable. In this paper, the main geometric parameters of curved bridges are studied. The finite strip method of analysis applied to folded plate structures as a box girder, for which a general computer program was written. This method of analysis is restricted to structures simply supported along their straight radial edges.

Finally, on the basis of this refined analytical method, the behaviour of curved box girder bridges is studied. In particular, wheel load distribution characteristics are investigated especially with respect to the load position of the bridges.

Keywords: Curved bridges, finite strip method, folded plate, box girder, simply supported.

1. Introduction
The choice of materials and geometry of structural elements basically is based on the several structural performance required, such as strength, stiffness and stability. After all the said criteria satisfied, then other considerations such as economic, aesthetics, and other non-technical aspects will be considered.

On the use of thin walled structures, stiffness of the structural elements depends mostly on the geometry of its cross section, for example folded plate. This type of structures was introduced in the 1930s in Germany, and then widely used in other countries in Europe. After the World War II, it was introduced in the United States when Winter and Pei in 1947 formulated the stress distribution for the first time. From that time on, the development of the theory, analysis and application of thin walled structures was getting momentum, especially in the US [6].

Various methods to analyse the folded plate structures have been developed in early 60s [13], whereby a modified version of what-so-called Gaafar method has been recommended, although later on it was found that it was not suitable for computerization. Moreover, the method has limitation for cases with small span-width ratio. Goldberg and Leve in 1957 developed the elasticity method, that was then applied by de Fries-Skene and Scordelis in 1964 [8] as stiffness approach. This method was able to analyse structures with small span, however it was still too complicated for

The finite element method was established by Zienkewicz and Cheung around 1965, and it was Rocky and Evans who reported the use of this method to study the behaviour of folded plate for the first time [6]. This method is found to be the most powerful tool in analysing the internal actions, and can be applied to analyse folded plate structures under various loading condition, type of supports and variation on the thickness of the plate. However, for structures with simple geometric configuration and boundary conditions, analysis using the finite element method may not be needed. For this reason, a simpler method has been developed by utilising long elements or strips, thus known as the Finite Strip Method.

Cheung has introduced the use of the finite strip approach to analyse the folded plate structures in 1969 [2], followed by the box girder in 1971 [3]. He combined the bending strip and plane stress strip to form the shell strip. The lower order finite strip for shell was then applied to analyse the orthotropic folded plate and eccentric folded plate in 1970 [3]. Higher order finite strip element with one internal nodal line was introduced by Loo and Cusens in 1971 [10]. This approach was then used by Cheung and Cheung in 1971 to analyse cut conical shape structures [4,5,6].

The objective of this study was to elaborate the behaviour of curved box girder bridges due to load as a function of horizontal gyration. Besides, this study was also aimed to reveal the characteristics of wheel load distribution on the curved box girder bridges due to various loading positions in accordance to Indonesian Loading Code for Bridges and Highways [15]. For these purposes, the Finite Strip Method was applied.

2. Methodology

The methods applied for this study was a computer simulation, whereby the computer program was developed based on the finite strip method. Two computer programs were written to include:

a. Analysis of box-girder bridges with straight geometry by applying both low- and high-order strips

b. Analysis of box-girder bridges with curved geometry by applying only the low-order strip.

The validity of the first program was verified by comparing the results from the simulation with the published data. For the second program, the validity of the program was evaluated by analysing box-girder bridges with straight geometry by introducing very large curvature and very small curved angle.

3. Analysis of Box Girder

Box girder can be assumed as combination of bending plates and plane-stress plates. However, in the elastic linear analysis, the two systems may be analysed independently (un-coupled). Structural Stiffness matrix and Force matrix can be obtained by assembling those obtained from the flexural analysis and from the plane stress analysis [7,9].

![Figure 1. Typical of the Bridge Box-girder Strip](image)
The $m^{th}$ term for the plate in the simply supported box girder can be written as:

$$[S]_{mn} \{\delta\}_m = \{F\}_m$$  

(1)

Where:

$$[S]_m = \begin{bmatrix} [S_{uu}]_p & [0] & [S_{uy}]_p & [0] \\ [0] & [S_{uu}]_p & [0] & [S_{uy}]_p \\ [S_{yj}]_p & [0] & [S_{yy}]_p & [0] \\ [0] & [S_{yj}]_p & [0] & [S_{yy}]_p \end{bmatrix}$$

$$\{\delta\}_m = [U_i, V_i, w_i, \theta_i, u_j, w_j, \theta_j]_m^T$$

$$\{F\}_m = [X_i, Y_i, Z_i, M_i, X_j, Y_j, Z_j, M_j]_m^T$$

For the whole structure, the equation can be constructed by assembling contribution from each plate:

$$\sum_{m=1}^{r} [S]_{mn} \{\delta\}_m = \{F\}_m$$  

(2)

Matrix equation as shown in Equation 1 is written in the local coordinate, whereby different plates may use different local coordinate systems. To assemble, it will be easier if all of them are transformed into the same coordinate system, called global coordinate system. Equation 1 is to be transformed into the global coordinate system. Transformation is carried out before the assembling process. Once the global displacement matrix found, it can be easily transformed back into the local coordinate system, to determine the local displacement matrix as well as stresses.

### 3.1 Curved Box Girder [1, 12, 14]

Generally, a curved box-girder is consisted of:

- a. Top and bottom flanges, which are horizontally curved-plates
- b. Exterior and interior curved webs, which are parts of cut conical shell structure

In the analysis, it is assumed that the boundaries of the structure are two curves and two radial planes on its supports.
3.2 Analysis of Curved Plate
Curved plate can be analysed by applying coordinate transformation from Cartesian coordinate into Polar coordinate system.

![Figure 3. Curved Plate Strip](image)

3.3 Analysis of Bending Curved Plate
Displacement function of curved strip can be obtained by transforming the displacement function from the rectangular strip. Displacement function is simplified by using non-dimensional variables:

\[ b' = \frac{r_2 - r_1}{2} \]
\[ R = \frac{r - r_1}{b'} \]

So for the simply supported system, the displacement function can be written as:

\[ W = \sum_{m=1} \left[ \left( 1 - \frac{3}{4} R^2 + \frac{1}{4} R^4 \right) b \left( R - R^2 + \frac{R^3}{4} \right) \left( \frac{3}{4} R^2 - \frac{1}{4} R^3 \right) b' \left( \frac{R^3}{4} - \frac{R^2}{2} \right) \right] \begin{bmatrix} w_{1m} \\ \psi'_{1m} \\ w_{2m} \\ \psi'_{2m} \end{bmatrix} \sin \frac{m \pi \theta}{\alpha} \]  
\[ ...........(3) \]

3.4 Analysis of Plane Stress Curved Plate
The displacement function:

\[ U = \sum_{m=1} \left[ \left( 1 - \frac{R}{2} \right) \frac{R}{2} \right] \begin{bmatrix} u_{1m} \\ u_{2m} \end{bmatrix} \sin \frac{m \pi \theta}{\alpha} \]  
\[ .................(4) \]

\[ V = \sum_{m=1} \left[ \left( 1 - \frac{R}{2} \right) \frac{R}{2} \right] \begin{bmatrix} v'_{1m} \\ v'_{2m} \end{bmatrix} \cos \frac{m \pi \theta}{\alpha} \]  
\[ .................(5) \]
3.5 Analysis of Cut Conical Shell

In the analysis of the cut conical shell, flexural action and plane stress action cannot be analysed separately (coupled) because of the curved surface (Figure 4).

\[ U = [(1 - \bar{x}) u_{1m} + (\bar{x}) u_{2m}] \sin \frac{m \pi \theta}{\alpha} \] .................................................. (6)

\[ V = [(1 - \bar{x}) v_{1m} + (\bar{x}) v_{2m}] \cos \frac{m \pi \theta}{\alpha} \] .................................................. (7)

\[ W = [(1 - 3\bar{x}^2 + 2\bar{x}^3) w_{1m} + x(1 - 2\bar{x}^2 + \bar{x}) \psi_{1m} + (3\bar{x}^2 - 2\bar{x}^3) w_{2m} + x(\bar{x}^2 - \bar{x}) \psi_{2m}] \sin \frac{m \pi \theta}{\alpha} \] .............................. (8)

Where:
\[ \bar{x} = \frac{x}{2b} \]

4. Computer Programming and Simulation

A computer program called “FSTRIP.BAS” has been written in QBASIC to be used to analyse prismatic folded plate structures simply supported along its radial side. This program can be used to analyse box girder bridges as folded plate structures with straight or curved geometry.

The program consists of two sub-programs, i.e. “STRSTRIP.BAS” to analyse structures with straight geometric by applying lower and higher orders. The second one is “CURSTRIP.BAS”. It can be used to analyse structures with curved geometry. This sub-program can be applied also in analysing structures with straight geometry by introducing very large curved curvature and very small curved angle.
4.1 Validity of the program

Before the computer program can be applied to analyse bridge box girder, its validity has to be verified first. Two ways have been adopted, first was to compare the output from “STRRSTRIP.BAS” with data available from various reports and also from “SAP2000” software. The second approach was to compare the output from “CURRSTRIP.BAS” with output from “STRRSTRIP.BAS”, i.e. by analysing structures with straight geometry by giving very large curvature and very small curved angle.

From the results of verification, the computer programs showed very accurate output in analysing bridge box girder. The computer programs were found to be reliable for analyzing box girder bridges.

4.2 Application on Curved-Bridge-Box-Girder Structure

For a simply supported two-cell curved bridge box girder with radial distance \( r = 3000 \text{ cm} \), curved length \( l = 1200 \text{ cm} \) and ‘span’ of curved section \( \alpha = 0.4 \text{ rad} \), to be analysed using “CURRSTRIP.BAS” with two different loading truck positions in accordance to the 1987 Indonesian Loading Code for Bridges and Highways [15], \( E_x = E_y = 2 \times 10^5 \text{ kg/cm}^2 \); \( 
u_x = 
u_y = 0.15 \); \( G = 86956 \text{ kg/cm}^2 \) (Figure 5). In this case, it can be seen that truck A caused higher stress in longitudinal direction and maximum moment in transversal direction than truck B. This phenomenon can be explained that the exterior girder of the bridge is more flexible than the interior one.

Figure 5. Curved Box Girder Bridge
(a) Plan  
(b) Cross section I–I  
(c) Curved Strip and Numbering of Nodal Lines and Strips
Figure 6. Moment in the Transversal Direction at Mid-span

Figure 7. Stresses in the Longitudinal Direction at Mid-span
5. Conclusions

1. When the horizontal curvature of the bridge increased, the exterior girder becomes more flexible. When the exterior girder is loaded, load distribution is better with the increase in the horizontal curvature.

2. Interior girder is stiffer than the exterior one, thus it tends to carry more load. If it is loaded, interior girder tends to not distributing the load.

References


15. 1987 Indonesian Loading Code for Bridges and Highways